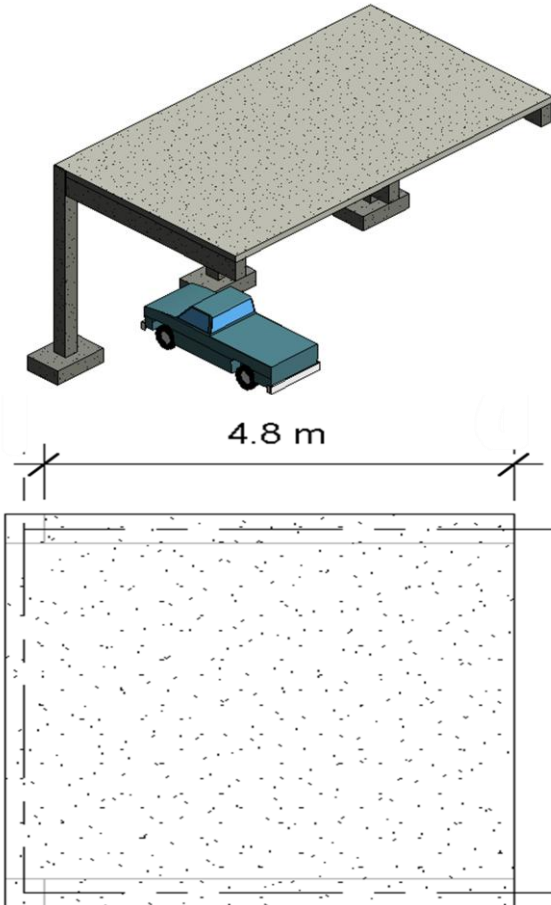


Typical Solutions

Note: use $f_c' = 25$ MPa and $f_y = 420$ MPa for all questions
Provide enough drawings to illustrate your answer for steel reinforcement.

Q1 (50%): check the adequacy of the beam shown below according to ACI requirement. $W_D = 2.5$ kN/m (including self-weight) and $W_L = 2.5$ kN/m



Solution:

1. Calculate $\rho = \frac{A_s}{bd}$

$$A_s = 3 \times \frac{\pi}{4} \times 16^2 = 603.2 \text{ mm}^2$$

$$d = 600 - 40 - 10 - \frac{16}{2} = 542 \text{ mm}$$

$$\rho = \frac{603.2}{300 \times 542} = 3.7 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.018$$

$$\rho < \rho_{max} \text{ O.k}$$

2. Calculate ϕ

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{603.2 \times 420}{0.85 \times 25 \times 300} = 39.74 \text{ mm}$$

$$c = \frac{a}{\beta_1} = \frac{39.74}{0.85} = 46.75 \text{ mm}$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{542 - 46.75}{46.75} \times 0.003 = 0.0317 > 0.005$$

$$\therefore \phi = 0.9$$

3. Calculate ϕM_n

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) = 0.9 \times 603.2 \times 420 \times \left(542 - \frac{39.74}{2} \right) \times 10^{-6}$$

$$\phi M_n = 119 \text{ kN.m}$$

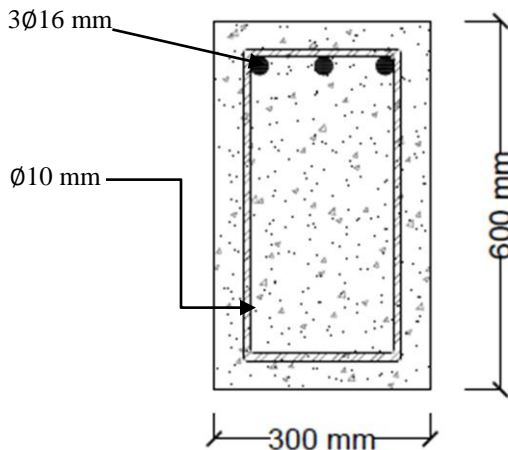
4. Find M_u and compare it with ϕM_n

$$W_D = 2.5 \text{ kN/m}, W_L = 2.5 \text{ kN/m}$$

$$W_u = 1.2W_D + 1.6W_L = 1.2 \times 2.5 + 1.6 \times 2.5 = 7 \text{ kN/m}$$

$$M_u = \frac{w_u \times \ell^2}{2} = \frac{7 \times 4.8^2}{2} = 80.64 \text{ kN.m} < \phi M_n$$

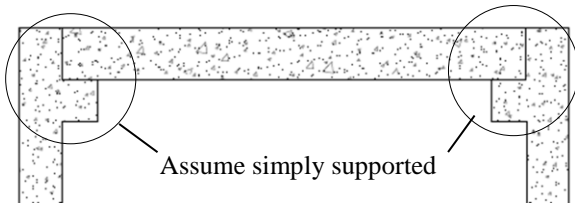
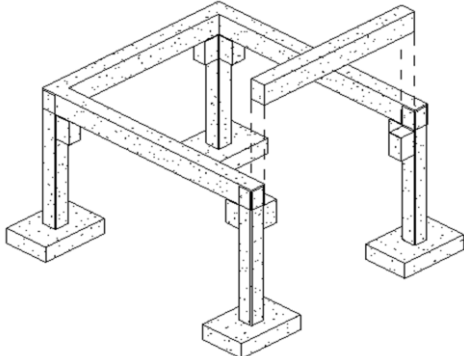
The section is O.k ■



Q2 (50%): Design a simply supported rectangular reinforced concrete beam shown in Figure below.

Assume that the designer intends to use:

- $M_u = 350 \text{ kN.m}$
- Use $\rho = 0.5 \rho_{max}$ and $\frac{d}{b} = 3$
- Rebar diameter 20mm for longitudinal reinforcement.
- Rebar diameter 10mm for stirrups.
- Two layers of reinforcement.



Solution:

1. Compute required factored applied moment M_u
 $M_u = 350 \text{ kN.m}$

2. $M_u = \phi M_n = \phi \rho f_y b d^2 (1 - 0.59 \frac{\rho f_y}{f_c})$

As it was mentioned in question $\rho = 0.5 \rho_{max}$ & $\frac{d}{b} = 3$

$$\rho_{max} = 0.85 \times \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \text{ and } \epsilon_u = 0.003$$

$$\rho_{max} = 0.85 \times 0.85 \times \frac{25}{420} \frac{0.003}{0.003 + 0.004} = 0.0184$$

$$\rho = 0.5 \rho_{max} = 0.5 \times 0.0184 = 9.2 \times 10^{-3}$$

3. Substitute both of selected ρ and ration of $\frac{d}{b}$ in the main equation:

$$M_u = \phi M_n = \phi \rho f_y b d^2 (1 - 0.59 \frac{\rho f_y}{f_c})$$

$$350 \times 10^6 = 0.9 \times 9.2 \times 10^{-3} \times 420 \times b d^2 (1 - 0.59 \times \frac{9.2 \times 10^{-3} \times 420}{25})$$

$$350 \times 10^6 = 3.16 \times b d^2$$

$$b d^2 = 110.75 \times 10^6 \text{ mm}^2$$

$$\text{As } \frac{d}{b} = 3 \rightarrow d = 3b$$

$$b = 230.87 \text{ mm} \approx 250 \text{ mm} \therefore d = 3 \times 250 = 750 \text{ mm}$$

4. Compute required steel area

$$A_{s_{required}} = \rho \times (b d) = 9.2 \times 10^{-3} \times 250 \times 750 = 1725 \text{ mm}^2$$

5. Compute the required number of bars (n)

$$\text{No. of bars (n)} = \frac{A_s}{A_{bar}} = \frac{1725}{\frac{\pi}{4} 20^2} = \frac{1725}{314} = 5.49 \approx 6$$

6. Check if rebars can in be put in single layer

$$b_{required} = 2 \times \text{cover} + 2 \times \text{stirrups diameter} + \text{No. of rebars} \times \text{bar diameter} + (\text{No. of rebars} - 1) \times \text{spacing between rebars}$$

$$b_{required} = 2 \times 40 + 2 \times 10 + 3 \times 20 + 2 \times 25 = 210 \text{ mm}$$

$$b_{required} = 210 < b_{provided} = 250 \text{ O.K}$$

7. Compute the depth (h)

$$h_{\text{for two layer}} = 750 + 40 + 10 + 12.5 + 20 = 832.5 \approx 835 \text{ mm}$$

Use (250 × 750) mm with 6Ø20 mm

8. Check the assumption of $\phi = 0.9$

$$a = \frac{A_s \cdot f_y}{0.85 f_c' \cdot b} = \frac{1884.9 \cdot 420}{0.85 \cdot 25 \cdot 250} = 149 \text{ mm}$$

$$c = \frac{a}{\beta_1} = \frac{149}{0.85} = 175.32 \text{ mm}$$

$$d_t = 835 - 40 - 10 - 10 = 775 \text{ mm}$$

$$\epsilon_t = \frac{d_t - c}{c} \epsilon_u = \frac{775 - 175.32}{175.32} \times 0.003 = 0.0102 > 0.005 \therefore \phi = 0.9$$

9. Draw final detailed section ■

